NAG Toolbox for MATLAB

f08kn

1 Purpose

f08kn computes the minimum norm solution to a complex linear least-squares problem

$$\min_x \|b - Ax\|_2.$$

2 Syntax

```
[a, b, s, rank, info] = f08kn(a, b, rcond, 'm', m, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f08kn uses the singular value decomposition (SVD) of A, where A is an m by n matrix which may be rank-deficient.

Several right-hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m by r right-hand side matrix B and the n by r solution matrix X.

The effective rank of A is determined by treating as zero those singular values which are less than **rcond** times the largest singular value.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

2: **b**(**ldb**,$*$) – **complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The m by r right-hand side matrix B.

3: **rcond** – **double scalar**

Used to determine the effective rank of A. Singular values $\mathbf{s}(i) \leq \mathbf{rcond} \times \mathbf{s}(1)$ are treated as zero. If $\mathbf{rcond} < 0$, ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrices B and X.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork, rwork

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The first $\min(m, n)$ rows of A are overwritten with its right singular vectors, stored row-wise.

2: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

b contains the n by r solution matrix X. If $m \geq n$ and $\mathbf{rank} = n$, the residual sum of squares for the solution in the ith column is given by the sum of squares of the modulus of elements $n+1, \ldots, m$ in that column.

3: **s($*$) – double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The singular values of A in decreasing order.

4: **rank – int32 scalar**

The effective rank of A, i.e., the number of singular values which are greater than $\mathbf{rcond} \times \mathbf{s}(1)$.

5: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **s**, 9: **rcond**, 10: **rank**, 11: **work**, 12: **lwork**, 13: **rwork**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

The algorithm for computing the SVD failed to converge; if **info** $= i$, i off-diagonal elements of an intermediate bidiagonal form did not converge to zero.

7 Accuracy

See Section 4.5 of Anderson *et al.* 1999 for details.

8 Further Comments

The real analogue of this function is f08ka.

9 Example

```
a = [complex(0.47, -0.34), complex(-0.4, +0.54), complex(0.6, +0.01),
complex(0.8, -1.02);
     complex(-0.32, -0.23), complex(-0.05, +0.2), complex(-0.26, -0.44),
complex(-0.43, +0.17);
      complex(0.35, -0.6), complex(-0.52, -0.34), complex(0.87, -0.11),
complex(-0.34, -0.09);
     complex(0.89, +0.71), complex(-0.45, -0.45), complex(-0.02, -0.57),
complex(1.14, -0.78);
      complex(-0.19, +0.06), complex(0.11, -0.85), complex(1.44, +0.8),
complex(0.07000000000000001, +1.14)];
b = [complex(-1.08, -2.59);
     complex(-2.61, -1.49);
     complex(3.13, -3.61);
     complex(7.33, -8.01);
     complex(9.119999999999999, +7.63)];
rcond = 0.01;
[aOut, bOut, s, rank, info] = f08kn(a, b, rcond)
```

```
aOut =
   -0.3352                0.3721 - 0.1084i   -0.0616 + 0.4811i   -0.4465 +
0.5503i
    0.5389                0.0529 - 0.2020i    0.0597 + 0.7312i    0.0289 -
0.3564i
   -0.3736                0.4361 + 0.5487i    0.3416 + 0.2185i    0.3581 -
0.2766i
   -0.6765               -0.3830 - 0.4102i   -0.1106 + 0.2234i    0.0465 -
0.4038i
   -0.0978 + 0.0134i   -0.2470 + 0.5898i   -0.1840 + 0.1355i    0.3393 -
0.6232i
bOut =
   1.1673 - 3.3222i
   1.3480 + 5.5028i
   4.1762 + 2.3434i
   0.6465 + 0.0105i
```

```
  -0.0749 + 0.1345i
s =
    2.9979
    1.9983
    1.0044
    0.0064
rank =
           3
info =
           0
```